The Effect of Ion Slip on the Flow of Reiner-Rivlin Fluid Due a Rotating Disk with Heat Transfer

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The magnetohydrodynamic rotating disk flow and heat transfer of a non-Newtonian Reiner-Rivlin conducting fluid is studied considering the ion slip. The governing nonlinear equations are solved numerically using finite differences. The results show that the inclusion of the ion slip and the non-Newtonian fluid characteristics have interesting effects on the velocity and temperature distributions.

Key Words : Rotating Disk Flow, Heat Transfer, Hydromagnetic Flow, Ion Slip Effect, Numerical Solution

1. Introduction

The boundary layer induced by a rotating disk of great scientific importance owing to itsa relevance to applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, viscometer, turbo-machinary, crystal growth processes, and chemical vapor deposition reactor. The hydrodynamic flow due to an infinite rotating disk was first formulated by von Karman (1921), then asymptotic solution for the governing equations was obtained by Cochran (1934) and Benton (1966), respectively, in the steady and unsteady state. Later, the rotating disk problem is studied by many authors under different physical conditions (Lee et al., 0000). Rotating disk flows of a electrically conducting fluids was studied by many researchers (Attia, 1998 ; Aboul-Hassan and Attia, 1997 ; Attia and Aboul-Hassan, 2001) without the ion slip. The steady rotating disk flow of a non-Newtonian fluid was considered in (Mithal, 1961 ; Srivastava, 1961). The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen (1952) for a variety of Prandtl numbers in the steady state. Sparrow and Gregg (1960) studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Later Attia (2003) extended the problem discussed in (Millsaps and Pohlhausen, 1952 ; Sparrow and Gregg, 1960) to the unsteady state in the presence of an applied uniform magnetic field and obtained a numerical solution for the relevant equations for any Prandtl number while the Hall current and ion slip were neglected. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is high or when the collision frequency is low (Crammer and Pai, 1973). Furthermore, the masses of the ions and electrons are different and, in turn, their motions will be different. Usually, the diffusion velocity of electrons is larger than that of ions and, as a first approximation, the electric current

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density is determined mainly by the diffusion velocity of the electrons. However, when the electromagnetic force is very large (such as in the case of strong magnetic field), the diffusion velocity of the ions may not be negligible (Crammer and Pai, 1973). If we include the diffusion velocity of ions as well as that of electrons, we have the phenomena of ion slip. In the above mentioned work, the Hall and ion slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable under these conditions, and the Hall current as well as the ion slip are important; they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic-force term (Crammer and Pai, 1973).

In the present work the unsteady magnetohydromagnetic (MHD) flow with heat transfer of an incompressible, non-Newtonian Reiner-Rivlin fluid, and electrically conducting fluid due to the uniform rotation of an infinite non-conducting disk in an axial uniform steady magnetic field is studied considering the ion slip. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Crammer and Pai, 1973). The governing non-linear differential equations are solved numerically using the finite difference approximations. Some interesting effects for the Hall current, the ion slip, and the non-Newtonian fluid characteristics on the velocity and temperature fields are reported.

2. Basic Equations

The disk is assumed to be insulating and rotating impulsively from rest in the $z=0$ plane about the z-axis with a uniform angular velocity ω. The fluid is assumed to be incompressible and has density ρ, kinematical viscosity ν, and electrical conductivity σ. An external uniform magnetic field is applied in the z-direction and has a constant flux density $\mathbf{B}_o$. The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect and the ion slip can not be neglected (Crammer and Pai, 1973). Due to the axial symmetry of the problem about the z-axis, the cylindrical coordinates (r, ϕ, z) are used. For the sake of definiteness, the disk is taken to be rotating in the positive ϕ direction. Due to the symmetry about the $z=0$ plane, it is sufficient to consider the problem in the upper half space only. The non-Newtonian fluid considered in the present paper is that for which the stress tensor τ_j^i is related to the rate of strain tensor e_j^i as (Mithal, 1961),

$$\tau_j^i = 2\mu e_j^i + 2\mu_c e_k^i e_j^k - p\delta_j^i,\ e_j^i = 0$$

where p is denoting the pressure, μ is the coefficient of viscosity and μ_c is the coefficient of cross viscosity. If the Hall and ion slip terms are retained in generalized Ohm's law, then the equations of unsteady motion in cylindrical coordinates (Crammer and Pai, 1973)

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{dz} = 0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) + \frac{\sigma B_o^2}{((1+\beta_e\beta_i)^2 + \beta_e^2)}((1+\beta_e\beta_i)u - \beta_e v) = \frac{\partial \tau_r^r}{\partial r} + \frac{\partial r_r^z}{\partial z} + \frac{\tau_r^r - \tau_\varphi^\varphi}{r} \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) + \frac{\sigma B_o^2}{((1+\beta_e\beta_i)^2 + \beta_e^2)}((1+\beta_e\beta_i)v - \beta_e u) = \frac{\partial \tau_\varphi^r}{\partial r} + \frac{\partial r_\varphi^z}{\partial z} + \frac{2\tau_\varphi^r}{r} \tag{3}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = \frac{\partial \tau_z^r}{\partial r} + \frac{\partial \tau_z^z}{\partial z} + \frac{\tau_z^r}{r} \tag{4}$$

where u, v, and w are the velocity components in the directions of increasing r, ϕ, and z, β_i is the ion slip parameter, $\beta_e = \sigma\beta B_o$ is the Hall parameter which can take positive or negative values, $\beta = 1/nq$ is the Hall factor, n is the electron concentration per unit volume, $-q$ is the charge of the electron (Crammer and Pai, 1973). Positive

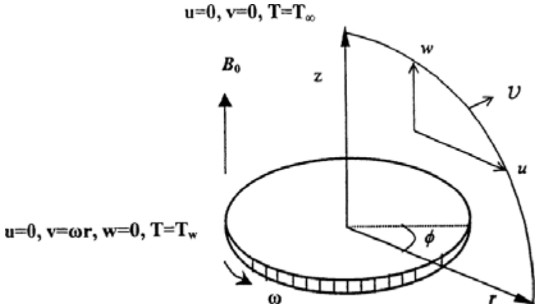


Fig. 1 Physical model and coordinate system

values of β_e mean that B_o is upwards and the electrons of the conducting fluid gyrate in the same sense as the rotating disk. For negative values of β_e, B_o is downwards and the electrons gyrate in an opposite sense to the disk. We introduce von Karman transformations (von Karman, 1921),

$$u=r\omega F,\ v=r\omega G,\ w=\sqrt{\omega v}H,$$
$$z=\sqrt{v/\omega}\zeta,\ p-p_\infty=-\rho v\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of the modified vertical coordinate ζ and t. We define the magnetic interaction number γ by $\gamma=\sigma B_o^2/\rho\omega$ which represents the ratio between the magnetic force to the fluid inertia force. With these definitions, Eqs. (1) ~ (4) take the form

$$\frac{\partial H}{\partial \zeta}+2F=0 \tag{5}$$

$$\frac{\partial F}{\partial t}-\frac{\partial^2 F}{\partial \zeta^2}+H\frac{\partial F}{\partial \zeta}+F^2-G^2$$
$$+\frac{\gamma}{((1+\beta_i\beta_e)^2+\beta_e^2)}((1+\beta_i\beta_e)F-\beta_e G) \tag{6}$$
$$-\frac{1}{2}K\left(\left(\frac{\partial F}{\partial \zeta}\right)^2+3\left(\frac{\partial G}{\partial \zeta}\right)^2+2F\frac{\partial^2 F}{\partial \zeta^2}\right)=0$$

$$\frac{\partial G}{\partial t}-\frac{\partial^2 G}{\partial \zeta^2}+H\frac{\partial G}{\partial \zeta}+2FG$$
$$+\frac{\gamma}{((1+\beta_i\beta_e)^2+\beta_e^2)}((1+\beta_i\beta_e)G+\beta_e F) \tag{7}$$
$$-K\left(\frac{\partial F}{\partial \zeta}\frac{\partial G}{\partial \zeta}-F\frac{\partial^2 G}{\partial \zeta^2}\right)=0$$

$$\frac{\partial H}{\partial t}-\frac{\partial^2 H}{\partial \zeta^2}+H\frac{\partial H}{\partial \zeta}+\frac{7}{2}K\frac{\partial H}{\partial \zeta}\frac{\partial^2 H}{\partial \zeta^2}-\frac{\partial P}{\partial \zeta}=0 \tag{8}$$

where K is the parameter which describes the non-Newtonian behavior, $K=\mu_c/\mu\omega$ and it represents the ratio of the shear stress due to viscous property to the normal stress accompanying with the elastic property of the fluid. The boundary conditions are given as

$$t=0,\ F=0,\ G=0,\ H=0 \tag{9a}$$

$$\zeta=0,\ F=0,\ G=1,\ H=0 \tag{9b}$$

$$\zeta\rightarrow\infty,\ F\rightarrow 0,\ G\rightarrow 0,\ P\rightarrow 0 \tag{9c}$$

The initial condition are given by Eq. (9a). Equation (9b) indicates the no-slip conditions of viscous flow applied at the surface of the disk. and ensures that the convective velocity normal to the surface of the disk specifies the mass injection or withdrawal. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (9c).

Due to the difference in temperature between the wall and the ambient fluid heat transfer takes place. The energy equation, by neglecting the dissipation terms, takes the form (Mithal, 1961),

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)-k\frac{\partial^2 T}{\partial z^2}=0 \tag{10}$$

where T is the temperature of the fluid, k and c_p are the thermal conductivity and the specific heat at constant pressure of the fluid. The boundary conditions for the energy problem are that the temperature, by continuity considerations, equals T_w at the surface of the disk. At large distances from the disk, $T\rightarrow T_\infty$ where T_∞ is the temperature of the ambient fluid.

In terms of the non-dimensional variable $\theta=(T-T_\infty)/(T_w-T_\infty)$ and using von Karman transformations, Eq. (10) takes the form,

$$\frac{\partial \theta}{\partial t}+\frac{1}{\Pr}\frac{d^2\theta}{d\zeta^2}+H\frac{d\theta}{d\zeta}=0 \tag{11}$$

where Pr is the Prandtl number given by, $\Pr=c_p\mu/k$. The boundary conditions in terms of □ are expressed as

$$t=0,\ \forall\zeta:\theta=0, \tag{12a}$$

$$\zeta=0:\theta=1,\ \zeta\rightarrow\infty,\ \theta\rightarrow 0 \tag{12b}$$

The significant velocity and temperature variations in the fluid are confined to the region adjacent to the disk which constructs viscous as well as thermal boundary layers. We define the thickness of these layers by certain standard measures (Sparrow and Gregg, 1960). The first of these is the displacement thickness. Since the radial flow is zero both at the disk surface and at infinity, a radial displacement thickness would have very little meaning. Then, for the tangential direction, we define a displacement thickness as (Sparrow and Gregg, 1960)

$$\delta_{dis}=\int_0^\infty G d\zeta$$

In physical terms, δ_{dis} gives the thickness of a fictitious layer of fluid which is rotating at a uniform tangential velocity $r\omega$ and is carrying a tangential mass flow equal to that carried by the actual tangential velocity distribution.

Also, as a measure of the extent of the thermal layer, we may introduce a thermal thickness based on the temperature excess ($T-T_\infty$) above the ambient fluid. Then,

$$\delta_t=\int_0^\infty \theta d\zeta$$

Physically speaking, δ_t is the thickness of a fictitious fluid layer at a temperature T_w whose integrated temperature excess over T_∞ is identical with that of the actual temperature distribution.

Numerical solution for the governing nonlinear Eqs. (5) ~ (7) with conditions given by Eq. (9), using the finite-difference, leads to a numerical oscillation problem resulting from the discontinuity between the initial and boundary conditions (9a) and (9b). The same discontinuity occurs between the initial and boundary conditions for the energy equation (see Eq. (12)). A solution for this numerical problem is achieved by using proper coordinate transformations, as suggested by Ames (1977) for similar problems. Expressing Eqs. (5) ~ (7) and (11) in terms of the modified coordinate $\eta=\zeta/2\sqrt{t}$ we get

$$\frac{\partial H}{\partial \zeta}+4\sqrt{t}F=0 \tag{13}$$

$$\frac{\partial F}{\partial t}-\frac{\eta}{2t}\frac{\partial F}{\partial \eta}-\frac{1}{4t}\frac{\partial^2 F}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{\partial F}{\partial \zeta}+F^2 -G^2+\frac{\gamma}{((1+\beta_i\beta_e)^2+\beta_e^2)}((1+\beta_i\beta_e)F-\beta_e G) -\frac{K}{8t}\left(\left(\frac{\partial F}{\partial \eta}\right)^2+3\left(\frac{\partial G}{\partial \eta}\right)^2+2F\frac{\partial^2 F}{\partial \eta^2}\right)=0 \tag{14}$$

$$\frac{\partial G}{\partial t}-\frac{\eta}{2t}\frac{\partial G}{\partial \eta}-\frac{1}{4t}\frac{\partial^2 G}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{\partial G}{\partial \eta}+2FG +\frac{\gamma}{((1+\beta_i\beta_e)^2+\beta_e^2)}((1+\beta_i\beta_e)G+\beta_e F) -\frac{K}{4t}\left(\frac{\partial F}{\partial \eta}\frac{\partial G}{\partial \eta}-F\frac{\partial^2 G}{\partial \eta^2}\right)=0 \tag{15}$$

$$\frac{\partial \theta}{\partial t}-\frac{\eta}{2t}\frac{\partial \theta}{\partial \eta}+\frac{1}{4t\,\mathrm{Pr}}\frac{\partial^2 \theta}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{\partial \theta}{\partial \zeta}=0 \tag{16}$$

Equations (13) ~ (16) represent coupled system of non-linear partial differential equations which are solved numerically under the initial and boundary conditions (9) and (12) using the finite difference approximations. A linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then Crank-Nicolson implicit method is used at two successive time levels (Ames, 1977). An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas-algorithm (Ames, 1977). Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the η-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and $\Delta\eta$ in time and space, respectively. The modified Eqs. (13) ~ (16) are integrated from $t=0$ to $t=1$. Then, the solution obtained at $t=1$ is used as the initial condition for integrating Eqs. (5) ~ (7) and (11) from $t=1$ towards the steady state.

The resulting system of difference equations has to be solved in the infinite domain $0<\zeta<\infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_\infty=10$ which is adequate for the ranges of the parameters studied here. Larger finite distances or smaller step size do not show any significant change in the results. Convergence of the scheme is assumed when all of the variables $F, G, H, \theta, \partial F/\partial\zeta, \partial G/\partial\zeta$ and $\partial\theta/\partial\zeta$ for the last two successive approximations differs from unity by less than 10^{-6} for all values of ζ in $0<\zeta<10$ and all t. It should be pointed that the steady state solutions reported by Attia and Aboul-Hassan (1997 ; 2001) when $K=0$ and $\gamma=0$. Also, the steady state solutions reported by Mithal (1961) and Srivastava (1961) are reproduced by setting $\gamma=0$ in the present results.

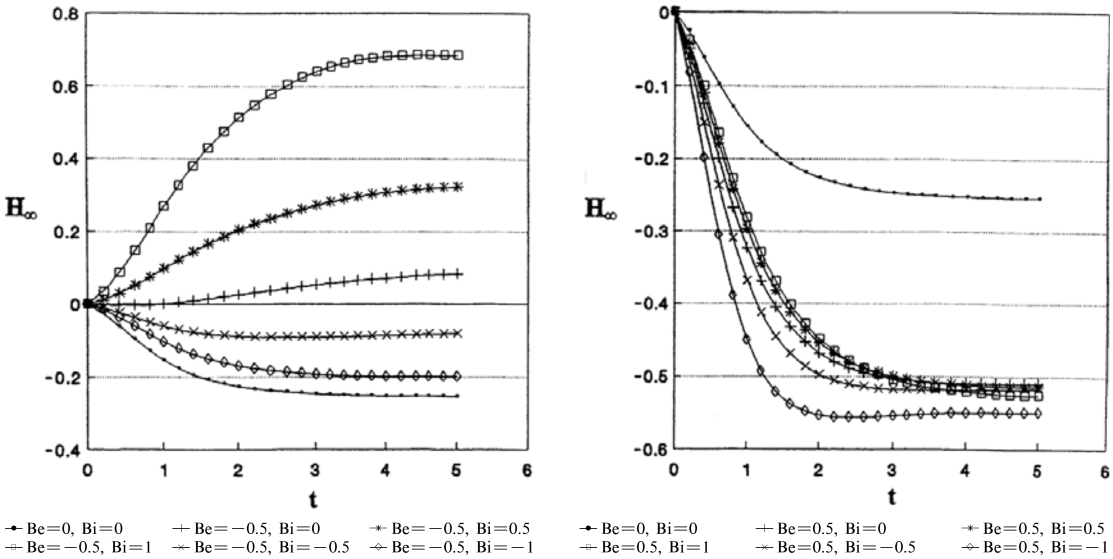
3. Results and Discussion

The three velocity components F, G, and H are obtained at different values of ζ. These velocity components have some general characteristics which can be predicted from the basic equations. The value of H at a given ζ decreases as F in the region below it increases. This follows from the continuity equation. Neglecting the Hall and ion slip effects ($\beta_e=0, \beta_i=0$), the applied uniform magnetic field (defined in terms of the parameter γ) represents the single effect of the magnetic field on the flow. It is clear from Eqs. (14) and (15) that the magnetic field has a damping effect on the radial and azimuthal velocity components due to the magnetic resistive forces. Equations (13) till then that the magnetic field has, in turn, a damping effect on the axial flow towards the disk.

The Hall parameter β_e appears in the magnetic force terms and its contribution, neglecting the ion slip ($\beta_i=0$), is proportional to $(F-\beta_e G)/(1+\beta_e^2)$ or $(G+\beta_e F)/(1+\beta_e^2)$. For small values of Be, the effect of Be on the numerator is stronger than its effect on the denominator. A small positive value of Be decreases the magnetic damping on F and increases the magnetic damping on G, thus increases F and decreases both H and G. A small negative value of Be decreases F and increases both H and G. For large positive values of β_e, the factor $(F-\beta_e G)$ may turn out to be negative and the magnetic field has a propelling effect on F, which may exceed its hydrodynamic value and thus the value of H is below its hydrodynamic value. For such large values of β_e, the effect on G is due mainly to the factor $1/(1+\beta_e^2)$ which becomes very small and produces an increase in G. For large negative values of Be, the argument is reversed. The magnetic damping on F is reduced to the decrease in $1/(1+\beta_e^2)$. Thus F increases but is still less than its hydrodynamic value, and consequently H decreases but is more than its hydrodynamic value. The factor $(G+\beta_e F)$ may become negative and this pushes G above its hydrodynamic value, thus the magnetic field has a propelling effect on G. For very large positive or negative values of β_e, the magnetic force term decreases much and the limit $\beta_e\rightarrow+\infty$ or $-\infty$ corresponds to the hydrodynamic limit.

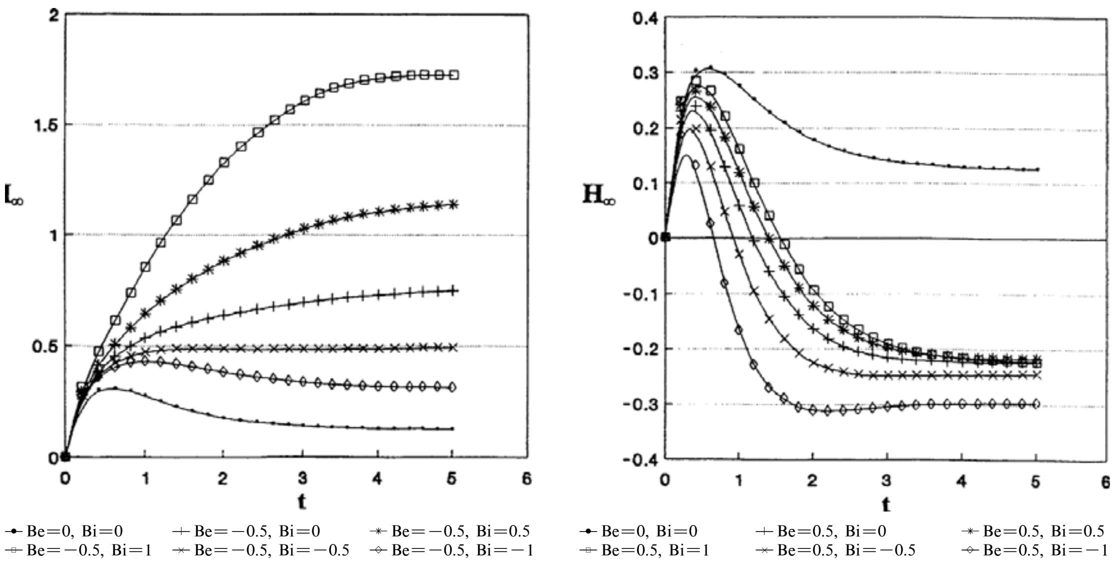
Considering the ion slip, the parameter $\alpha=(1+\beta_i\beta_e)$ appears in the magnetic force terms and its contribution is proportional to $(\alpha F-\beta_e G)/(\alpha^2+\beta_e^2)$ or $(\alpha G+\beta_e F)/(\alpha^2+\beta_e^2)$. For small positive values of α, the effect of α depends on the magnitude and sign of both parameters β_e and β_i. As the velocity component H increases, the axial flow towards the disk decreases which avoids bringing fluid at near-ambient temperature towards the surface of the disk. This decreases the heat transfer at the surface of the disk. It is clear that the heat transfer at the surface of the disk increases as H decreases.

Figures 2(a) and (b) present the time development of the axial velocity at infinity H_∞. For various values of the ion slip parameter β_i and for $\beta_e\leq 0$ and $\beta_e\geq 0$, respectively, in the case of $K=0$. Figure 2(a) shows that, for $\beta_e=-0.5$ and $\beta_i=0$, H_∞ reverses its direction at a certain distance from the disk. Increasing I, for $\beta_e<0$, increasing H_∞ and reverses its direction for all time as a result of increasing the effective conductivity $(=\sigma/((1+\beta_i\beta_e)^2+\beta_e^2))$ which increases the damping force on F and consequently increases H_∞. Figure 2(a) indicates also that for $\beta_e\beta_i>0$, increasing the magnitude of β_i decreases H_∞ due to the decrease in the effective conductivity which decreases the damping force on F and, in turn, increases F which decreases H_∞. Figure 2(b) indicates that when $\beta_e>0$ and $\beta_i>0$, increasing β_i increases H_∞ while for $\beta_i<0$, increasing the magnitude of β_i decreases H_∞. It is of interest to detect that the variation of H_∞ with β_i depends on t. Figures 3(a) and (b) present the time development of the axial velocity at infinity H_∞ for various values of the ion slip parameter β_i and for $\beta_e\le 0$ and $\beta_e\ge 0$, respectively, in the case $K=1$.



(a) $\beta_e\le 0$ (b) $\beta_e\ge 0$

Fig. 2 Time development of H_∞ for various values of β_i and $K=0$

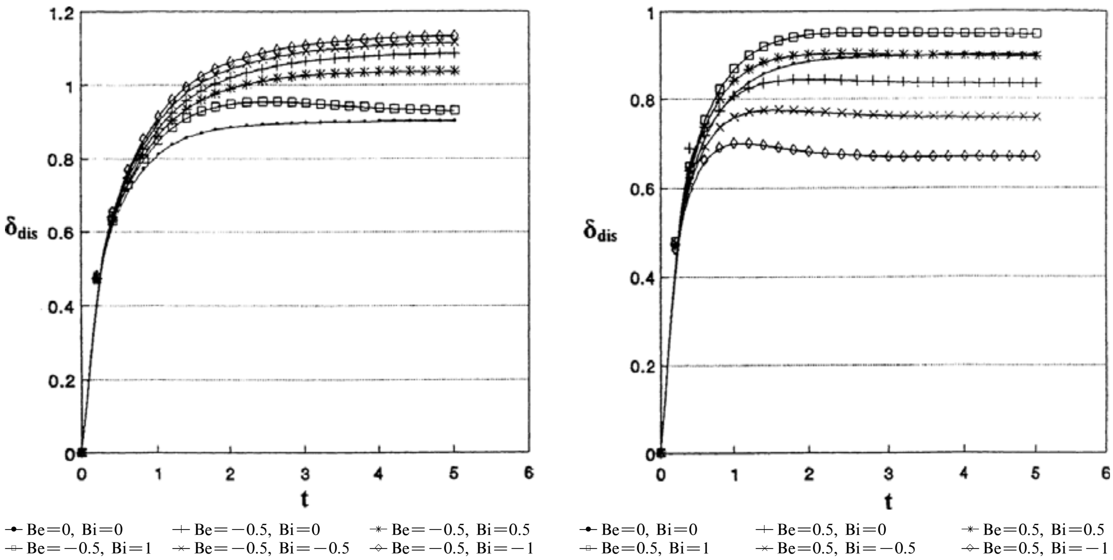


(a) $\beta_e\le 0$ (b) $\beta_e\ge 0$

Fig. 3 Time development of H_∞ for various values of β_i and $K=1$

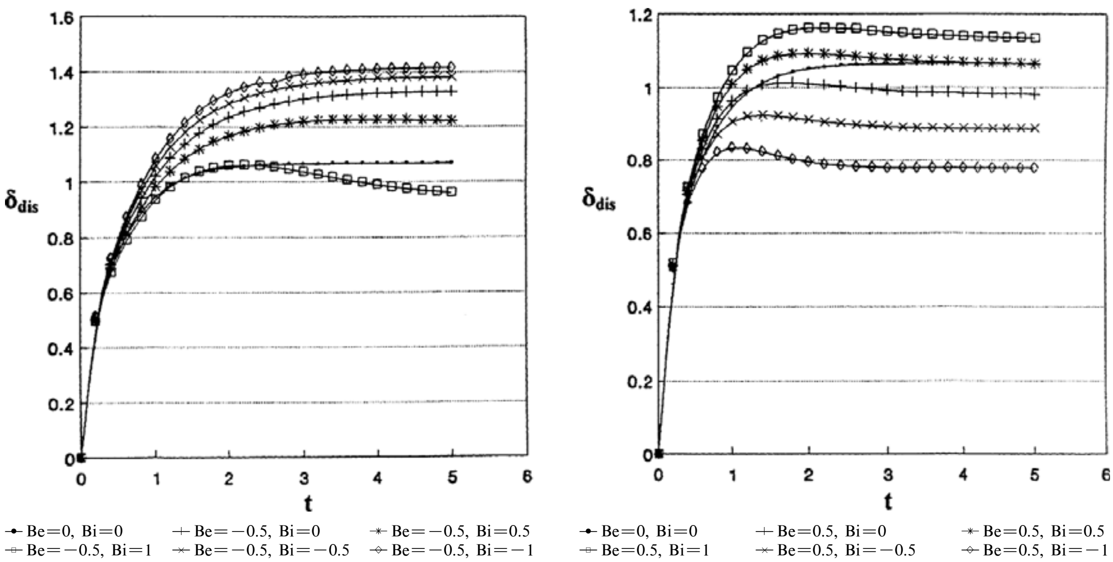
It is shown in the figure that increasing K decreases the axial flow towards the disk. Figure 3(a) presents an interesting effect of K in reversing the direction of the flow for all values of t and β_i. It is clear from Fig. 3(b) that the parameter K leads to the reversal of the direction of the axial flow towards the disk for some time only with the appearance of overshooting during the progression of time for all values of β_i. The effect of β_i on H_∞ is more pronounced for the non-Newtonian $K>0$.

Figures 4(a) and (b) present the time development of the tangential displacement thickness δ_{dis} for various values of the ion slip parameter β_i and for $\beta_e \leq 0$ and $\beta_e \geq 0$, respectively, in case $K=0$. As shown in Fig. 4(a), small negative values of



(a) $\beta_e \leq 0$ (b) $\beta_e \geq 0$

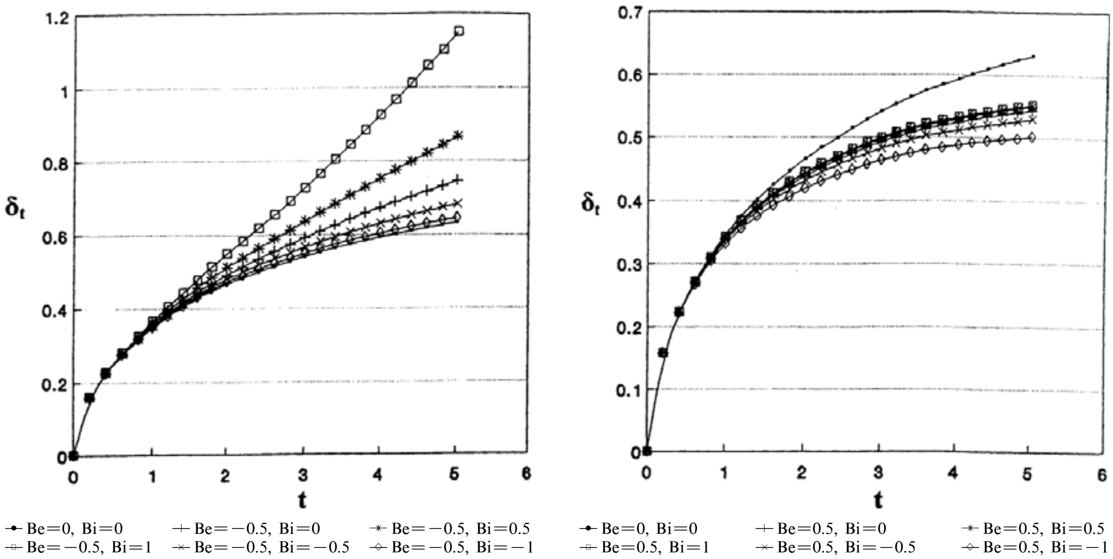
Fig. 4 Time development of δ_{dis} for various values of β_i and $K=0$



(a) $\beta_e \leq 0$ (b) $\beta_e \geq 0$

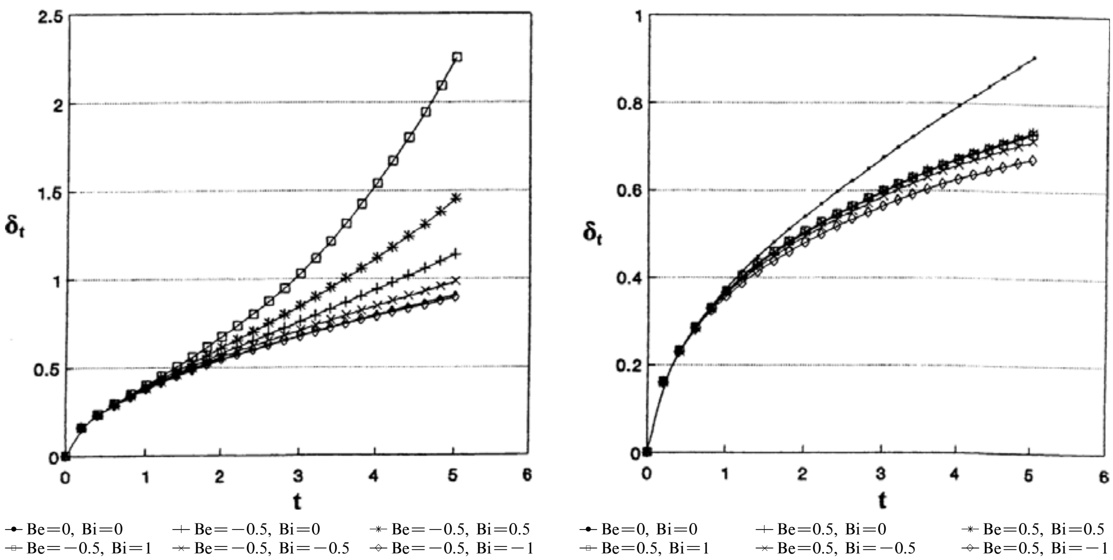
Fig. 5 Time development of δ_{dis} for various values of β_i and $K=1$

β_e increases δ_{dis} as a result of decreasing the magnetic damping. Increasing β_i, with $\beta_e<0$, decreases δ_{dis}, due to the increase in the effective conductivity, and results in the appearance of overshooting in δ_{dis} during time in case of large β_i. Figure 4(a) shows also that for negative values of β_i increasing the magnitude of β_i increases δ_{dis} due to the decrease in the damping force on G. Figure 4(b) describes the same findings. For $\beta_i \beta_e>0$, increasing β_i increases δ_{dis}, while for $\beta_i \beta_e<0$, increasing the magnitude of β_i decreases δ_{dis}. Figures 5(a) and (b) present the time development of the tangential displacement thickness δ_{dis} for various values of the ion slip parameter β_i and for $\beta_e\le 0$ and $\beta_e\ge 0$, respectively, in the case $K=1$. It is clear from the figure that as K



(a) $\beta_e\le 0$ (b) $\beta_e\ge 0$

Fig. 6 Time development of δ_t for various values of β_i and $K=0$



(a) $\beta_e\le 0$ (b) $\beta_e\ge 0$

Fig. 7 Time development of δ_t for various values of β_i and $K=1$

increases, δ_{dis} increases for all values of β_e and β_i. The effect of β_i on δ_{dis} is more pronounced for $K>0$.

Figure 6(a) and (b) present the time development of the thermal displacement thickness δ_t for various values of the ion slip parameter β_i and for $\beta_e \leq 0$ and $\beta_e \geq 0$, respectively, in the case $K=0$. As shown in Fig. 6(a), for $\beta_i=0$, increasing the magnitude of β_e, increases δ_t as a result of increasing the axial flow velocity which avoids bringing the fluid at near ambient temperature to the disk and hence increases the temperature. Increasing β_i increases the axial flow velocity and hence increases the temperature which increases δ_t. On the other hand, for $\beta_i<0$, increasing the magnitude of β_i decreases δ_t as a result of decreasing the axial flow velocity. Figure 6(b) shows that, β_i has a pronounced effect on δ_t only when $\beta_i\beta_e<0$. In this case, increasing the magnitude of β_i decreases the temperature and then decreases δ_t due to its effect in damping the axial flow towards the disk. Figure 7(a) and (b) present the time development of the thermal displacement thickness δ_t for various values of the ion slip parameter β_i and for $\beta_e \leq 0$ and $\beta_e \geq 0$, respectively, in the case $K=1$ and $\mathrm{Pr}=10$. The figures indicate the effect of increasing δ_t as a result of increasing K due to the influence of K in damping the axial flow towards the disk. The effect of β_i on δ_t is more pronounced for $K>0$.

4. Conclusions

The transient MHD flow and heat transfer of a non-Newtonian Reiner-Rivlin fluid due to an infinite rotating disk are studied in the presence of a uniform magnetic field perpendicular to its plane with the ion slip. The inclusion of the Hall effect, the ion slip and the non-Newtonian fluid characteristics reveals some interesting phenomena and it is found that the signs of the Hall and ion slip parameters are important. It is of interest to find, in the non-Newtonian case, that the axial flow reverses direction for all time and for $\beta_e<0$ while, for $\beta_e>0$, it reverses direction during time with the appearance of overshooting. Also, it is found that the influence of the parameter β_i on the axial flow and the heat transfer is more apparent for the non-Newtonian case than the Newtonian case.

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